14. Panel and Clustered Data

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Gov 2002 (Harvard)

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- Focus up until now on iid data, but often doesn't hold.
- **Panel** and **clustered** data are two common non-iid data.
- Panel data also holds hope for removing unmeasured heterogeneity.

1/ Panel Data

Motivation

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- If have data on countries over time, can we make any progress in spite of these problems?

Ross data

```r
library(tidyverse)
library(haven)
ross <- read_dta("../assets/ross-democracy.dta")
ross <- ross |>
  filter(!is.na(kidmort_unicef), !is.na(democracy), !is.na(GDPcur)) |>
  group_by(id) |>
  filter(var(democracy, na.rm = TRUE) > 0)
head(ross[,c("cty_name", "year", "democracy", "infmort_unicef")])
```

```
## # A tibble: 6 x 4
##   cty_name   year democracy infmort_unicef
##   <chr>     <dbl>     <dbl>          <dbl>
## 1 Albania    1990         0             36
## 2 Albania    1995         1             30
## 3 Argentina  1970         0             59
## 4 Argentina  1980         0             33
## 5 Argentina  1990         1             25
## 6 Argentina  1995         1             22
```

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- **Time series, cross-sectional (TSCS) data**: smaller n, large T (a political science term, mostly)

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- Assume that if we could measure c_i, we would have the correct CEF:

$$\mathbb{E}[u_{it} \mid \mathbf{X}_{it}, c_i] = 0 \quad \Longrightarrow \quad \mathbb{E}[Y_{it} \mid \mathbf{X}_{it}, c_i] = \mathbf{X}_{it}'\boldsymbol{\beta} + c_i$$

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    2. Errors might be correlated with the covariates
- Both problems arise out of ignoring the **unmeasured heterogeneity** inherent in c_i

Pooled OLS with Ross data

```r
library(lmtest)
library(sandwich)
pooled.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur),
                 data = ross)
coeftest(pooled.mod, vcov = vcovHC)
```

```
##
## t test of coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.3338     0.6279   16.46  < 2e-16 ***
## democracy    -0.5639     0.1135   -4.97  1.3e-06 ***
## log(GDPcur)  -0.2486     0.0287   -8.66  7.7e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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- Pooled OLS will be inconsistent for the CEF parameters, $\boldsymbol{\beta}$.

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- Two approaches that leverage repeated observations:
    - **Differencing** look at changes over time.
    - **Fixed effects** look at relationships within units.

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&= (\mathbf{X}_{i2}' - \mathbf{X}_{i1}')\boldsymbol{\beta} + (c_i - c_i) + (u_{i2} - u_{i1}) \\
&= \Delta \mathbf{X}_i'\boldsymbol{\beta} + \Delta u_i
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- Under these assumptions, pooled OLS on the differences is consistent.

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- Key fact: mean of the time-constant c_i is just c_i

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- **Fixed effects model**: another way to remove unmeasured heterogeneity
- Focuses on **within-unit comparisons**: changes in Y_{it} and X_{it} relative to their within-group means
- First note that taking the average of the Y's over time for a given unit leaves us with a very similar model:

$$\begin{aligned}
\overline{Y}_i &= \frac{1}{T}\sum_{t=1}^{T}\left[\mathbf{X}_{it}'\boldsymbol{\beta} + c_i + u_{it}\right] \\
&= \left(\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_{it}'\right)\boldsymbol{\beta} + \frac{1}{T}\sum_{t=1}^{T}c_i + \frac{1}{T}\sum_{t=1}^{T}u_{it} \\
&= \overline{\mathbf{X}}_i'\boldsymbol{\beta} + c_i + \overline{u}_i
\end{aligned}$$

- Key fact: mean of the time-constant c_i is just c_i
- This regression is sometimes called the “between regression”

Within transformation

- **Fixed effect** or **within transformation**:

$$(Y_{it} - \overline{Y}_i) = (\mathbf{X}'_{it} - \overline{\mathbf{X}}'_i)\boldsymbol{\beta} + (u_{it} - \overline{u}_i)$$

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- If we write $\ddot{Y}_{it} = Y_{it} - \overline{Y}_i$, then we can write this more compactly as:

$$\ddot{Y}_{it} = \ddot{\mathbf{X}}_{it}'\boldsymbol{\beta} + \ddot{u}_{it}$$

Fixed effects with Ross data

```
library(fixest)
fe.mod <- fixest::feols(
  log(kidmort_unicef) ~ democracy + log(GDPcur) | id,
  data = ross, vcov = "hetero")
summary(fe.mod)
```

```
## OLS estimation, Dep. Var.: log(kidmort_unicef)
## Observations: 237
## Fixed-effects: id: 53
## Standard-errors: Heteroskedasticity-robust
##              Estimate Std. Error t value     Pr(>|t|)
## democracy      -0.156     0.0314   -4.97 0.0000015379 ***
## log(GDPcur)    -0.354     0.0252  -14.03    < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 0.18124     Adj. R2: 0.95396
##                 Within R2: 0.711842
```

Strict exogeneity

$$\ddot{Y}_{it} = \ddot{\mathbf{X}}'_{it}\boldsymbol{\beta} + \ddot{u}_{it}$$

- To use OLS on demeaned data, need $\mathbb{E}[\ddot{\mathbf{X}}_{it}\ddot{u}_{it}] = 0$.

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$$\mathbb{E}[u_{it}|\mathbf{X}_{i1}, \mathbf{X}_{i2}, \ldots, \mathbf{X}_{iT}, c_i] = 0$$

  - u_{it} uncorrelated with all covariates for unit i at any point in time.
  - Rules out lagged dependent variables, since $Y_{i,t-1}$ is a function of $u_{i,t-1}$.

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  - R/Stata and the like will drop it from the regression.
  - Any time-constant variable gets "absorbed" by the fixed effect.
- Can include interactions between time-constant and time-varying variables, but lower order term of the time-constant variables get absorbed by fixed effects too.

Time-constant variables

- Pooled model with a time-constant variable, proportion Islamic:

```
library(lmtest)
p.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam, data = ross)
coeftest(p.mod, vcov = vcovHC)
```

```
##
## t test of coefficients:
##
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.36014    0.58133   17.82  < 2e-16 ***
## democracy   -0.47634    0.09441   -5.05  9.6e-07 ***
## log(GDPcur) -0.25597    0.02671   -9.58  < 2e-16 ***
## islam        0.00855    0.00106    8.06  5.2e-14 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Time-constant variables

- FE model, where the islam variable drops out, along with the intercept:

```
fe.mod2 <- feols(
  log(kidmort_unicef) ~ democracy + log(GDPcur) + islam | id,
  data = ross, vcov = "hetero")
summary(fe.mod2)
```

```
## OLS estimation, Dep. Var.: log(kidmort_unicef)
## Observations: 220
## Fixed-effects: id: 45
## Standard-errors: Heteroskedasticity-robust
##             Estimate Std. Error t value    Pr(>|t|)
## democracy     -0.144     0.0347   -4.14 0.000054978
## log(GDPcur)   -0.360     0.0257  -14.00   < 2.2e-16
##
## democracy   ***
## log(GDPcur) ***
## ... 1 variable was removed because of collinearity (islam)
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 0.185449     Adj. R2: 0.949078
##                  Within R2: 0.717818
```

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$$Y_{it} \text{ on } \mathbf{X}_{it}, D_{i2}, D_{i3}, \ldots D_{in}$$

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- Comments:
    - Pros: easy to implement and gives correct SEs.
    - Con: computationally slow with large n.
    - Usually better to use dedicated software like `fixest` package in R.

Example with Ross data

```r
lsdv.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur) + id,
               data = ross)
coeftest(lsdv.mod, vcov = vcovHC)[1:6,]
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   11.385     0.6306   18.05 2.01e-42
## democracy     -0.156     0.0366   -4.27 3.14e-05
## log(GDPcur)   -0.354     0.0295  -11.99 8.65e-25
## idARG          1.263     0.1425    8.87 6.82e-16
## idARM          0.462     0.1287    3.59 4.20e-04
## idBEN          1.334     0.0884   15.08 7.10e-34
```

```r
coeftest(fe.mod)[1:2,]
```

```
##             Estimate Std. Error z value Pr(>|z|)
## democracy     -0.156     0.0314   -4.97 6.69e-07
## log(GDPcur)   -0.354     0.0252  -14.03 1.08e-44
```

4/ Clustering

Clustered dependence: intuition

- Think back to the Gerber, Green, and Larimer (2008) social pressure mailer example.

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- Zero conditional mean error holds here (random assignment)
- Violation of **iid/random sampling**:
    - errors of individuals within the same household are correlated.
    - SEs are going to be wrong.
- Called **clustering** or **clustered dependence**

Clustered dependence: notation

- Clusters (groups): $g = 1, \dots, m$

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- Units are (usually) belong to a single cluster:
    - voters in households
    - individuals in states
    - students in classes
    - rulings in judges
- Outcome varies at the unit-level, Y_{ig} and the main independent variable varies at the cluster level, X_g.

Clustered dependence: example model

$$\begin{aligned} Y_{ig} &= \beta_0 + X_g\beta_1 + v_{ig} \\ &= \beta_0 + X_g\beta_1 + c_g + u_{ig} \end{aligned}$$

- u_{ig} unit error component with $\mathbb{V}[u_{ig}|X_g] = \sigma^2_u$

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- c_g cluster error component with $\mathbb{V}[c_g|X_g] = \sigma_c^2$
- c_g and u_{ig} are assumed to be independent of each other.
    - $\rightsquigarrow \mathbb{V}[v_{ig}|X_g] = \sigma_c^2 + \sigma_u^2$
- What if we ignore this structure and just use v_{ig} as the error?

Lack of independence

- Covariance between two units i and s in the same cluster:

$$\text{Cov}[v_{ig}, v_{sg}] = \sigma_c^2$$

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$$\text{Cor}[v_{ig}, v_{sg}] = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_u^2} = \rho_c$$

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- Zero covariance of two units i and s in different clusters g and k:

$$\text{Cov}[v_{ig}, v_{sk}] = 0$$

Example covariance matrix

- $\mathbf{v}' = \begin{bmatrix} v_{1,1} & v_{2,1} & v_{3,1} & v_{4,2} & v_{5,2} & v_{6,2} \end{bmatrix}$

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$$\mathbb{V}[\mathbf{v}|\mathbf{X}] = \begin{bmatrix} \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \sigma_c^2 & 0 & 0 & 0 \\ \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & 0 & 0 & 0 \\ \sigma_c^2 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \sigma_c^2 \\ 0 & 0 & 0 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 \\ 0 & 0 & 0 & \sigma_c^2 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 \end{bmatrix}$$

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- Variance matrix under i.i.d.:

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$$Y_{ig} = \beta_0 + X_g\beta_1 + c_g + u_{ig}$$

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- We can write the OLS estimator as:

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{g=1}^{m} \mathbb{X}'_g\mathbb{X}_g\right) \left(\sum_{g=1}^{m} \mathbb{X}'_g\mathbf{Y}_g\right)$$

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$$\left(\mathbb{E}[\mathbb{X}_g'\mathbb{X}_g]\right)^{-1}\mathbb{E}[\mathbb{X}_g'\mathbf{v}_g\mathbf{v}_g'\mathbb{X}_g]\left(\mathbb{E}[\mathbb{X}_g'\mathbb{X}_g]\right)^{-1}$$

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- With small-sample adjustment (reported by most software):

$$\widehat{\mathbf{V}}_{\hat{\boldsymbol{\beta}}}^{\text{CL1}} = \frac{m}{m-1}\frac{n-1}{n-k} \left(\mathbb{X}'\mathbb{X}\right)^{-1} \left(\sum_{g=1}^{m} \mathbb{X}_g'\hat{\mathbf{v}}_g\hat{\mathbf{v}}_g'\mathbb{X}_g\right) \left(\mathbb{X}'\mathbb{X}\right)^{-1}$$

Example: Gerber, Green, Larimer

Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	______
9995 JENNIFER KAY SMITH		Voted	______
9997 RICHARD B JACKSON		Voted	______
9999 KATHY MARIE JACKSON		Voted	______

Social pressure model

```
load("../assets/gerber_green_larimer.RData")
library(lmtest)
social$voted <- 1 * (social$voted == "Yes")
social$treatment <- factor(
  social$treatment,
  levels = c("Control", "Hawthorne", "Civic Duty", "Neighbors", "Self")
)
mod1 <- lm(voted ~ treatment, data = social)
coeftest(mod1)
```

```
##
## t test of coefficients:
##
##                     Estimate Std. Error t value
## (Intercept)          0.29664    0.00106  279.53
## treatmentHawthorne   0.02574    0.00260    9.90
## treatmentCivic Duty  0.01790    0.00260    6.88
## treatmentNeighbors   0.08131    0.00260   31.26
## treatmentSelf        0.04851    0.00260   18.66
##                     Pr(>|t|)
## (Intercept)          < 2e-16 ***
## treatmentHawthorne   < 2e-16 ***
## treatmentCivic Duty  5.8e-12 ***
## treatmentNeighbors   < 2e-16 ***
## treatmentSelf        < 2e-16 ***
## ---
```

Social pressure model, CRSEs

```r
library(sandwich)
coeftest(mod1, vcov = sandwich::vcovCL(mod1, cluster = social$hh_id))
```

```
##
## t test of coefficients:
##
##                        Estimate Std. Error t value
## (Intercept)             0.29664    0.00131  226.52
## treatmentHawthorne      0.02574    0.00326    7.90
## treatmentCivic Duty     0.01790    0.00324    5.53
## treatmentNeighbors      0.08131    0.00337   24.13
## treatmentSelf           0.04851    0.00330   14.70
##                        Pr(>|t|)
## (Intercept)             < 2e-16 ***
## treatmentHawthorne      2.8e-15 ***
## treatmentCivic Duty     3.2e-08 ***
## treatmentNeighbors      < 2e-16 ***
## treatmentSelf           < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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- Consistency of the CRSE are in the number of groups, not the number of individuals
    - CRSEs can be incorrect with a small ($<$ 50 maybe) number of clusters